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GENERALIZED SLAB MODEL AND FINITE LARMOR RADIUS EFFECTS

University of Rochester

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13. ABSTRACT

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The combined influence of finite Larmor radius corrections and inertia on the stability of the infinite slab model is studied. A simple dispersion relation is obtained.

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GENERALIZED SLAB MODEL AND FINITE LARMOR RADIUS EFFECTS

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I. INTRODUCTION

In previous semi-annual technical reports, we have developed a three-dimensional model of the equilibrium (i.e. pre-striation) growth of a barium cloud in the presence of a conducting background ionosphere. Early studies[1,2] considered the case of a highly conducting background with complete short-circuiting. In this case there is no polarization of the cloud and a diffusing elliptical gaussian density results. Later work[3,4] extended these results to include finite background conductivity with polarization effects and steepening of the backside.

Stability calculations[5] have been carried out (in the case of the highly conducting background) by studying the growth of small perturbations about the equilibrium developed above. These numerical calculations[4] have shown that the fastest growing eigenmode is concentrated on the rear side of the cloud. Calculational difficulties have not yet enabled one to establish the stability threshold (or whether one actually exists) nor has it yet yielded a grid-independent shape of the eigenmode. Further work on this point is in progress.

Meanwhile, some information can be gleamed from slab models, although there are troublesome questions concerning the validity of application to real three-dimensional clouds. The E X B instability was first derived in 1963 using such a model by Simon[6] and by Hoh[7] with $\vec{E}$ parallel to the density gradient. This analysis yielded a threshold value of E and a critical wavelength which was the first mode to go unstable (and the fastest growing at any later "linear" stage). Linson and Workman extended this analysis to include a component of the field perpendicular to the density gradient. In their treatment, they take the limit of collisionless electrons ab initio with the result that there is no threshold for instability onset and all wavelengths grow at about the same rate. This result is disturbing because it seems at variance with observed time-delay for striation onset and because the analysis gives no guide for predicting the size of observed striations.

There are a number of effects which could restore a threshold to the slab models above. These include effects of ion inertia and finite larmor radius which are discussed in Sect. III below. Another effect is the internal equilibrium electric field that developes in the plasma. In

the infinite slab model, such as that of Linson and Workman, this necessarily points towards the center of the slab in order to keep ions from diffusing out faster than electrons. This field direction is destabilizing on the basis of the Simon-Hoh model. In Sect. II, we develop a model of a slab with finite extent along the magnetic field and allow for a background of arbitrary conductivity. The resulting field can either point in or point out depending on the degree of short-circuiting by the background. A stability analysis of this more general slab model is now in progress.

II. SLAB MODEL IN A FINITE CONDUCTING BACKGROUND

The model we consider is sketched in Figure 1. The cloud is represented by a slab of finite extent along the magnetic field. We shall assume no variation of the equilibrium quantities in the y direction. $\nabla \times \underset{\sim}{E} = 0$ then implies $E_y =$ const. $= E_{oy}$

As shown in a previous report[3], if we expand the current divergence equation

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{J} = 0 \tag{1}$$

in the small parameter $D_{\parallel}^{+}/D_{\parallel}^{-}$ we obtain in the zeroth order approximation, the electric field

$$\underset{\sim}{E}^{(0)} = -A \frac{\nabla N}{N} - \nabla g(r_{\perp}) \tag{2}$$

where N is the equilibrium density.

To determine $g(r_{\perp})$ we shall integrate eqn. (1) along a magnetic field line from one boundary of the ionosphere to the other boundary. Using the condition that the current vanishes at the boundaries, we have

$$\int_{-L}^{L} \underset{\sim}{\nabla}_{\perp} \cdot \underset{\sim}{J}_{\perp} \, dz = 0 \tag{3}$$

In the large $(\Omega T)_{-}$ limit

$$\frac{1}{e} \underset{\sim}{\nabla}_{\perp} \cdot \underset{\sim}{J}_{\perp} = -D_{\perp}^{+} \nabla_{\perp}^{2} N + \mu_{\perp} \underset{\sim}{\nabla}_{\perp} (N \underset{\sim}{E}_{\perp}) - \mu_{\perp}^{+} (\Omega T)_{+}^{-1} \underset{\sim}{b} \cdot \nabla N \times \underset{\sim}{E} \tag{4}$$

where the symbols all have the same meaning as in previous reports.

If we substitute eqn. (2) into eqn. (4) and integrate the equation along a field line, we find

$$-\left(D_{\perp}^{+}+A\mu_{\perp}^{+}\right)\frac{\partial^{2}\widetilde{N}_{c}}{\partial x^{2}}-\mu_{\perp}^{+}\frac{\partial}{\partial x}\left(\widetilde{N}_{c}\frac{\partial \varphi}{\partial x}\right)-\mu_{\perp}^{+}(\Omega\tau)_{+}^{-1}E_{oy}\frac{\partial\widetilde{N}_{c}}{\partial x}$$
$$-\frac{1}{e}\Sigma_{A}^{P}\frac{\partial^{2}\varphi}{\partial x^{2}} = 0 \tag{5}$$

In arriving at eqn. (5) we set $N=N_c+N_A$ where N_A is the ambient density, and have defined

$$\widetilde{N}_{c}=\int_{-\ell/2}^{\ell/2}N_{c}\,dz$$

$$\Sigma_{A}^{P}=e\int_{-L}^{L}\mu_{\perp}^{+}N_{A}\,dz$$

We also assume that $(\Omega\tau)_{+}$ is independent of z within the length of the cloud.

Integrating eqn. (5), we find

$$-\left(D_{\perp}^{+}+A\mu_{\perp}^{+}\right)\frac{\partial\widetilde{N}_{c}}{\partial x}-\left(\mu_{\perp}^{+}\widetilde{N}_{c}+\frac{\Sigma_{A}^{P}}{e}\right)\frac{\partial\varphi}{\partial x}$$
$$-\mu_{\perp}^{+}(\Omega\tau)_{+}^{-1}E_{oy}\widetilde{N}_{c} = \text{const.}$$

To evaluate the constant, we consider a field line far away from the cloud. Where $\widetilde{N}_{c}=0$, $N=N_A$ and $\frac{\partial\varphi}{\partial x}=-E_{cx}$ we find

$$\text{const.} = \frac{1}{e}\Sigma_{A}^{P}E_{cx}$$

We have then

$$\frac{\partial g}{\partial x} = -\left(\frac{D_\perp^i}{\mu_\perp^i} + A\right)(1-\alpha)\frac{1}{\tilde{N}_c}\frac{\partial \tilde{N}_c}{\partial x} - \alpha E_{ox} - (1-\alpha)\frac{E_{oy}}{(\Omega\tau)_+} \tag{6}$$

where

$$\alpha(x) = \Sigma_A^P / [\, e\mu_\perp^i \tilde{N}_c(x) + \Sigma_A^P \,] \tag{7}$$

is the ratio of Pederson conductivity integrated along a magnetic field line not passing through the cloud to that integrated along a field line through the cloud.

It follows from eqns. (2) and (6) that

$$E_x^{(0)} = -A\frac{1}{N}\frac{\partial N}{\partial x} + [(D_\perp^i/\mu_\perp^i) + A](1-\alpha)\frac{1}{\tilde{N}_c}\frac{\partial \tilde{N}_c}{\partial x}$$
$$+ \alpha E_{ox} + (1-\alpha)\frac{E_{oy}}{(\Omega\tau)_+} \tag{8}$$

If the background ionosphere is assumed to be infinitely conducting, then $\alpha \to 1$ and we have

$$E_x^{(0)} = -A\frac{1}{N}\frac{\partial N}{\partial x} + E_{ox}$$

in agreement with the electric field used in earlier reports (cf. eqn. (9) of Ref. 3).

If the cloud's contribution dominates in the expression for the integrated conductivity, then $\alpha \to 0$. As the background density is negligible we have $\frac{1}{N}\frac{\partial N}{\partial x} = \frac{1}{\tilde{N}_c}\frac{\partial \tilde{N}_c}{\partial x}$ and then

$$E_x^{(0)} = \frac{D_\perp^i}{\mu_\perp^i}\frac{1}{N}\frac{\partial N}{\partial x} + \frac{E_{oy}}{(\Omega\tau)_+}$$

This is the electric field used in the slab model of Linson and Workman[8].

The expression for the electric field to the next order in the expansion in the small parameter $D_{\parallel}^{+}/D_{\parallel}^{-}$ is obtained by iteration from eqns. (1) and (2). We find

$$\frac{\partial}{\partial z}(NE_z^{(1)}) = C\frac{\partial^2 N}{\partial x^2} + K\frac{\partial}{\partial x}\left(N\frac{\partial g}{\partial x}\right) - LE_{oy}\frac{\partial N}{\partial x} \tag{9}$$

where

$$C = [D_{\perp}^{+} - D_{\perp}^{-} + A(\mu_{\perp}^{+} + \mu_{\perp}^{-})]/(\mu_{\parallel}^{+} + \mu_{\parallel}^{-})$$

$$K = (\mu_{\perp}^{+} + \mu_{\perp}^{-})/(\mu_{\parallel}^{+} + \mu_{\parallel}^{-})$$

$$L = [(\Omega\tau)_{+}\mu_{\perp}^{+} - (\Omega\tau)_{-}\mu_{\perp}^{-}]/(\mu_{\parallel}^{+} + \mu_{\parallel}^{-})$$

To find the equilibrium density, we substitute eqn. (2) into the ion continuity eqn. and obtain

$$\frac{\partial N}{\partial t} - (D_{\perp}^{+} + A\mu_{\perp}^{+})\frac{\partial^2 N}{\partial x^2} - (D_{\parallel}^{+} + A\mu_{\parallel}^{+})\frac{\partial^2 N}{\partial z^2}$$
$$- \mu_{\perp}^{+}\frac{\partial}{\partial x}\left(N\frac{\partial g}{\partial x}\right) + \mu_{\perp}^{+}(\Omega\tau)_{+}E_{oy}\frac{\partial N}{\partial x} = 0 \tag{10}$$

Making use of eqn. (6) we have

$$\frac{\partial N}{\partial t} - (D_{\parallel}^{+} + A\mu_{\parallel}^{+})\frac{\partial^2 N}{\partial z^2} - (D_{\perp}^{+} + A\mu_{\perp}^{+})\frac{\partial^2 N}{\partial x^2}$$
$$+ (D_{\perp}^{+} + A\mu_{\perp}^{+})\frac{\partial}{\partial x}\left[(1-\alpha)\frac{N}{\tilde{N}_c}\frac{\partial \tilde{N}_c}{\partial x}\right]$$
$$+ \mu_{\perp}^{+}\left[E_{ox} - \frac{E_{oy}}{(\Omega\tau)_{+}}\right]\frac{\partial}{\partial x}(\alpha N) + \frac{c}{B}E_{oy}\frac{\partial N}{\partial x} = 0 \tag{11}$$

If the ambient density is negligible inside the cloud, then $N \doteq N_c$. We obtain the solution for N_c

$$N_c = \tilde{N}_c \frac{1}{\sqrt{4\pi \mathcal{D}_{\parallel} t}} \exp(-z^2/4\mathcal{D}_{\parallel} t) \tag{12}$$

where $\mathcal{D}_{\parallel} = D^+_{\parallel} + A\mu^+_{\parallel}$

and $\tilde{N}_c$ is a solution of

$$\frac{\partial \tilde{N}_c}{\partial t} - (D^+_\perp + A\mu^+_\perp)\frac{\partial}{\partial x}\left(\alpha \frac{\partial \tilde{N}_c}{\partial x}\right) + \mu^+_\perp \left[E_{0x} - \frac{E_{0y}}{(\Omega\tau)_+}\right]\frac{\partial}{\partial x}(\alpha \tilde{N}_c) + \frac{c}{B}E^0_y \frac{\partial \tilde{N}_c}{\partial x} = 0 \tag{13}$$

In the case of an infinitely conducting background, $\alpha = 1$, we have

$$\frac{\partial \tilde{N}_c}{\partial t} - (D^+_\perp + A\mu^+_\perp)\frac{\partial^2 \tilde{N}_c}{\partial x^2} + \mu^+_\perp E_{0x}\frac{\partial \tilde{N}_c}{\partial x} + \mu^+_\perp (\Omega\tau)_+ E_{0y}\frac{\partial \tilde{N}_c}{\partial x} = 0$$

The solution for an instantaneous line source is

$$\tilde{N}_c = \frac{S_0}{\sqrt{4\pi \mathcal{D}_\perp t}} \exp\left[-(x - V_c t)^2/4\mathcal{D}_\perp t\right] \tag{14}$$

where

$$\mathcal{D}_\perp = D^+_\perp + A\mu^+_\perp$$

$$V_c = \mu^+_\perp \left[E_{0x} + (\Omega\tau)_+ E_{0y}\right]$$

We see that the strong short-circuiting will allow the cloud to diffuse at the ion rate and the cloud as a whole moves with the ion drift velocity in the ambient electric field.

In the case of dominant cloud conductivity

Eqn. (13) becomes

$$\frac{\partial \tilde{N}_c}{\partial t} + \frac{c}{B}E_{0y}\frac{\partial \tilde{N}_c}{\partial x} = 0$$

and then

$$\widetilde{N}_c = \widetilde{N}_{c_{t=0}} (x - U_0 t) \tag{15}$$

The cloud as a whole moves with the velocity $U_0 = cE_{0y}/B$. Had we included electron collisions, the density profile would also diffuse at the electron rate.

III. ION INERTIA AND FINITE LARMOR RADIUS EFFECT

The results in both references 1 and 8 show that there is no threshold for instability in the limit of collisionless electrons and with no variation of the perturbation in the magnetic field direction. This cannot be generally correct in the limit of collisionless ions as well, since in that case one can remove the constant external electric field by a uniform frame change with no alteration of the plasma dynamics. It is well known that the inclusion of previously omitted inertial terms in the equations of motion is one resolution of this paradox. However, the quantitative threshold for instability onset is too low to account for cloud observations.

Another mechanism with a similar effect is inclusion of finite Larmor radius correction in the fluid equations of motion. For completeness, and because some of the techniques are unfamiliar, we have calculated the linear dispersion relation with both included.

Roberts and Taylor[9] have shown that one can include the finite Larmor radius effect by proper modification of the ion pressure tensor in the magnetohydrodynamic equation. The ion momentum equation including inertia and finite Lar-

mor radius takes the form

$$nM\frac{d\underset{\sim}{V}^+}{dt} = -KT\nabla n + ne\underset{\sim}{E} + ne\frac{\underset{\sim}{V}^+\times\underset{\sim}{B}}{c} - \frac{nM\underset{\sim}{V}^+}{\tau} + \nu\underset{\sim}{\lambda} \qquad (16)$$

where

$$\nu = \frac{1}{4}a^2\Omega$$

$$\lambda_x = \frac{\partial}{\partial x}\left[nM\left(\frac{\partial V_y}{\partial x}+\frac{\partial V_x}{\partial y}\right)\right] - \frac{\partial}{\partial y}\left[nM\left(\frac{\partial V_x}{\partial x}-\frac{\partial V_y}{\partial y}\right)\right]$$

$$\lambda_y = -\frac{\partial}{\partial y}\left[nM\left(\frac{\partial V_y}{\partial x}+\frac{\partial V_x}{\partial y}\right)\right] - \frac{\partial}{\partial x}\left[nM\left(\frac{\partial V_x}{\partial x}-\frac{\partial V_y}{\partial y}\right)\right]$$

where a is the ion Larmor radius.

The ratio of the finite Larmor radius term to the collisional term is $\Omega\tau(a^2/\ell^2)$ where ℓ is some characteristic length. We shall limit our discussion to a slab model with no short-circuiting $(\alpha=0)$ and uniform in the z direction. The equilibrium ion velocities are

$$V_x^+ = -D_\perp^+\frac{1}{N}\frac{\partial N}{\partial x} + \mu_\perp^+ E_x + (\Omega\tau)_+\mu_\perp^+ E_y$$

$$= \frac{cE_{oy}}{B} = U_o$$

$$V_y^+ = \mu_\perp^+ E_y - (\Omega\tau)_+\left(-D_\perp^+\frac{1}{N}\frac{\partial N}{\partial x} + \mu_\perp^+ E_x\right) = 0$$

where we have inserted the equilibrium electric field obtained in the previous section. Note that the equilibrium velocities are constant in time and uniform in space. It is, therefore, consistent to omit the inertial and the finite Larmor radius terms in the equilibrium momentum equation.

The linearized momentum equation, in the frame moving with velocity U_o, and with $\chi = \frac{n_1}{N}$ is:

$$-i\omega \underset{\sim}{V}_1^+ = -\frac{KT}{M} i\underset{\sim}{k}\chi - i\underset{\sim}{k}\frac{e}{M}\phi + \frac{e}{MC}\underset{\sim}{V}_1^+ \times \underset{\sim}{B} - \frac{\underset{\sim}{V}_1^+}{\tau} + \nu\underset{\sim}{\delta} \quad (17)$$

Here $\delta_x = -\frac{1}{d} ik_y V_{1x}^+ - (k^2 + \frac{1}{d} ik_x) V_{1y}^+$

$\delta_y = (k^2 + ik_x \frac{1}{d}) V_{1x}^+ - ik_y \frac{1}{d} V_{1y}^+$

In arriving at eqn. (17), we assumed that $\frac{1}{d} = -\frac{1}{N}\frac{\partial N}{\partial X}$ is constant and Fourier analyzed the perturbed quantities. It is not difficult to solve for V_{1x}^+ and V_{1y}^+ in terms of χ and ϕ and substitute the expressions into the linearized ion continuity equation in the primed coordinates. We find

$$\left\{-i\omega + \left[\omega\left(\frac{k_x}{d} - ik^2\right) + \frac{1}{d}\left(\frac{ik_x}{\tau} + ik_y\Omega\right) + \frac{k^2}{\tau}\right]\frac{KT}{M}\frac{1}{D}\right\}\chi$$

$$+\left[\omega\left(\frac{k_x}{d} - ik^2\right) + \frac{k^2}{\tau} + \frac{1}{d}\left(\frac{ik_x}{\tau} + ik_y\Omega\right)\right]\frac{e}{M}\frac{1}{D}\phi = 0 \quad (18)$$

here $D = \left[-i\omega + \frac{1}{\tau} + ik_y\frac{\nu}{d}\right]^2 + \left[\Omega - \nu\left(k^2 + \frac{ik_x}{d}\right)\right]^2$

The inertia term and finite Larmor radius effect is negligible in the electron equation of motion. The linearized electron equation is easily seen to be

$$\left\{-i\omega - ik_y\frac{c}{B}\frac{E_{oy}}{\Omega\tau} + ik_y\frac{KT}{e}\frac{c}{B}\frac{1}{d}\right\}\chi$$

$$+ ik_y\frac{1}{d}\frac{c}{B}\phi = 0 \quad (19)$$

From eqns. (18) and (19) we obtain the dispersion relation

$$\omega\left[\frac{k_y}{\Omega d}\left\{\left(\frac{1}{\tau}-i\omega+ik_y\frac{\nu}{d}\right)^2+\left[\Omega-\nu\left(k^2+\frac{ik_x}{d}\right)\right]^2-\Omega^2\right\}\right.$$
$$\left.+\left(ik^2-\frac{k_x}{d}\right)\left(\frac{1}{\tau}-i\omega\right)\right]$$
$$+k_y\frac{U_0}{\Omega\tau}\left[\left(ik^2-\frac{k_x}{d}\right)\left(\frac{1}{\tau}-i\omega\right)-\frac{k_y\Omega}{d}\right]=0 \tag{20}$$

In the limit of vanishing Larmor radius $a=0$ (hence $\nu=0$) and with the assumption $|\omega|\tau\ll 1$, eqn. (20) becomes:

$$\omega\left[\frac{k_y}{\Omega d\tau^2}+\frac{1}{\tau}\left(ik^2-\frac{k_x}{d}\right)\right]$$
$$+k_y\frac{U_0}{\Omega\tau}\left[\frac{1}{\tau}\left(ik^2-\frac{k_x}{d}\right)-\frac{k_y\Omega}{d}\right]=0$$

Therefore

$$\omega=k_yU_0\,\frac{\Omega\tau k_y+k_x-ik^2d}{k_y-k_x\Omega\tau+ik^2d\Omega\tau} \tag{21}$$

In this limit we have the result of Linson and Workman[8] when proper Doppler shift in ω is allowed for between the moving frame and the neutral rest frame.

If we take the collisionless limit of eqn. (20) we find

$$\omega\left\{\frac{k_y}{\Omega d}\left[-\left(k_y\frac{\nu}{d}-\omega\right)^2-2\Omega\nu\left(k^2+\frac{ik_x}{d}\right)+\nu^2\left(k^2+\frac{ik_x}{d}\right)^2\right]\right.$$
$$\left.-i\omega\left(ik^2-\frac{k_x}{d}\right)\right\}=0$$

Note that the dependence on U_0 vanishes. If we neglect small terms of order $(ak)^2$ and $(a/d)^2$ or higher, the above equation divided by k_y^2 reduces to

$$\omega\left\{\omega\left[\frac{1}{k_y^2}\left(k^2+\frac{ik_x}{d}\right)-\frac{\omega}{k_y d\Omega}\right]-\frac{2\nu}{k_y d}\left(k^2+\frac{ik_x}{d}\right)\right\}=0 \qquad (22)$$

There are two low frequency modes ($\omega \ll \Omega$). One has $\omega = 0$ and the other is a purely oscillating mode with $\omega \cong 2\nu k_y/d$. The third root has a high frequency of order $k_y d\Omega$ and is not relevant to the present discussion.

If we define the equivalent gravitational force $g = -U_0/\tau$ and then take the collisionless limit of eqn. (20), neglecting small terms of order $(ak)^2$, $(a/d)^2$ and ω/Ω we have the dispersion relation for waves propagation perpendicular to the density gradient $(k_x = 0)$

$$\omega^2 - \left(2\nu k_y \frac{1}{d} - \frac{g k_y}{\Omega}\right)\omega + \frac{g}{d} = 0 \qquad (23)$$

This also agrees with that obtained by Roberts and Taylor[9] when the difference in the sign of ω is allowed for. Threshold estimates are now being investigated.

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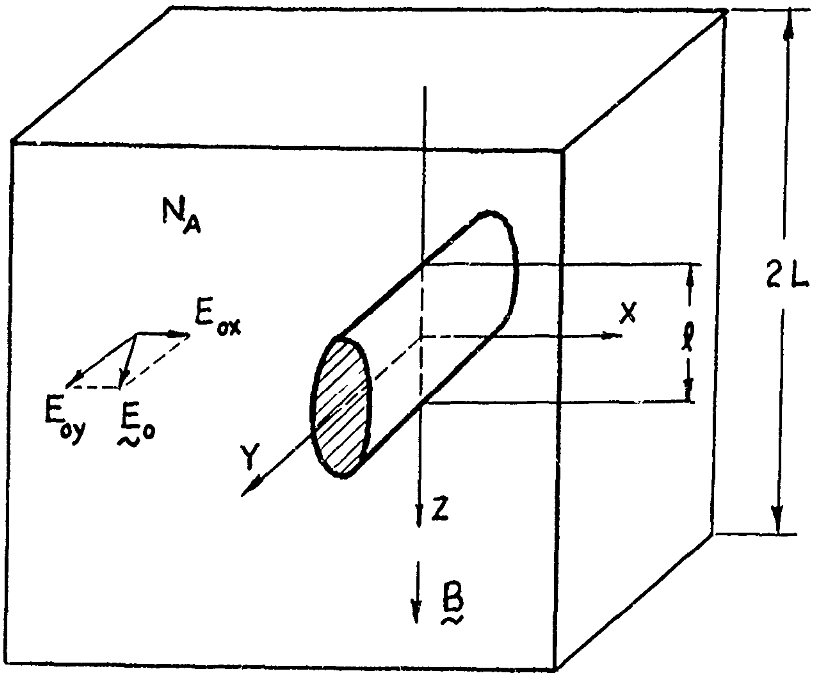

FIG. I FINITE SLAB IN A MAGNETIC FIELD